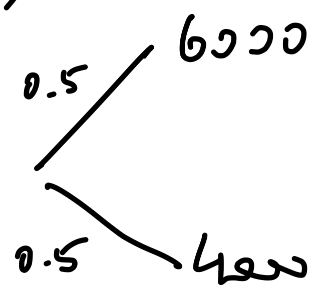
FIN 532

Problem Set 2

(1) a)

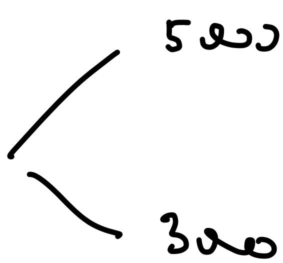


Gamble

$$E\,u(W) = 0.5 \ln 6000 + 0.5 \ln 4000$$
$$= \ln CE$$
$$CE = \exp(0.5 \ln 6000 + 0.5 \ln 4000)$$
$$= 4{,}898.98$$
$$\pi_i = 5000 - 4898.98 = 101.2$$

The insurance premium is 101.2. Thus, paying 125 for insurance is too much.

b)



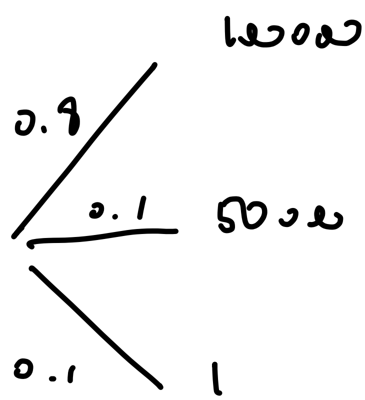
New gamble

$$CE = \exp(0.5 \ln 5000 + 0.5 \ln 3000)$$
$$= 3872.98$$

$$\pi_i = 127.02$$

For this new level of wealth, it makes sense to pay 125 for insurance.

(2)



$$CE = \exp(0.8 \ln 100\,000 + 0.1 \ln 50000 + 0.1 \ln 1)$$
$$= 29\,505.09$$

$$\pi_i = 100\,000 - 29\,505.09 = 70\,494.91$$

This might seem high, but you are fully insuring some pretty big risks that happen with high probability.

(3) The expected payoff of both gambles is

$$E(A) = 0.09 \times (-2) + 0.3 \times 4 + 0.4 \times 10 + 0.21 \times 16$$

$$= 8.38$$

$$E(B) = 0.09 \times (-2) + 0.3 \times (0.5 \times 3 + 0.5 \times 5)$$

$$+ 0.4 \times 10 + 0.21 \times (1/3 \times 12 + 2/3 \times 18)$$

$$= 8.38$$

The variances are

$$V(A) = 0.09 (-2 - 8.38)^2 + 0.3 \times (4 - 8.38)^2$$

$$+ 0.4 (10 - 8.38)^2 + 0.21 (16 - 8.38)^2$$

$$= 28.70$$

$$V(B) = 0.09 (-2 - 8.38)^2 + 0.15 (3 - 8.38)^2$$

$$+ 0.15 (5 - 8.38)^2 + 0.4 (10 - 8.38)^2$$

$$+ 0.07 \times (12 - 8.38)^2 + 0.14 (18 - 8.38)^2$$

$$= 30.68 > 28.70.$$

Intuitevely, it does not make sense to choose a gamble that pays the same on average but has higher variance.

Optional ← If you want a formal proof.

More formally, consider the following table

Prob	A	ε	$A+\varepsilon = B$
0.09	-2	0	-2
0.15	4	-1	3
0.15	4	1	5
0.4	10	0	10
0.07	16	-4	12
0.21	16	$+2$	18

Note the

$$E(\varepsilon) = 0.15(-1) + 0.15(1) + 0.07(-4) + 0.14(2)$$
$$= 0$$

but $V(\varepsilon) > 0$.

By Jensen's inequality conditioning on A.

$$u[E(A+\varepsilon \mid A)] > E(u(A+\varepsilon) \mid A)$$

$$\Rightarrow \quad u(A) > E(u(A+\varepsilon) \mid A)$$

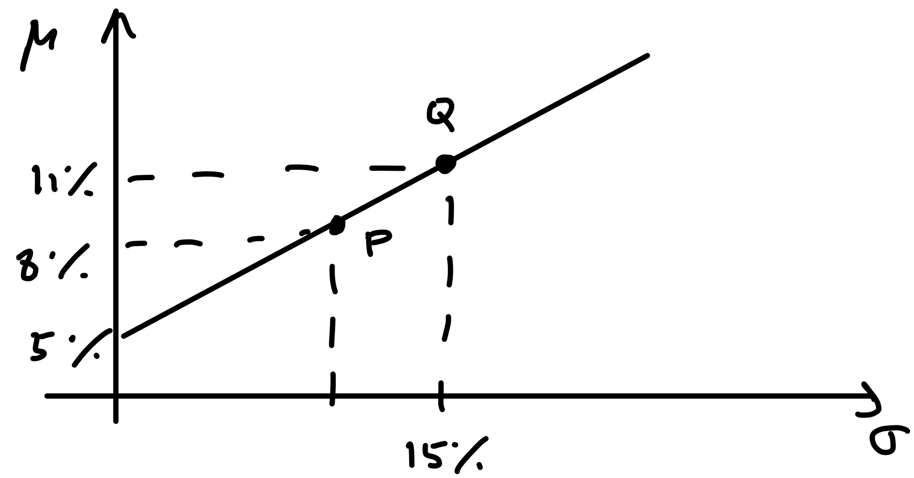
$$\Rightarrow \quad E(u(A)) > E(E(u(A+\varepsilon) \mid A))$$

$$\Rightarrow \quad E\,u(A) > E\,u(A+\varepsilon)$$

$$\Rightarrow \quad E\,u(A) > E\,u(B)$$

for $u(\cdot)$ concave $\Leftrightarrow$ risk averse investor.

(4) a.



$$\mu_P = (1-W)\,0.05 + W\,0.11 = 0.08$$

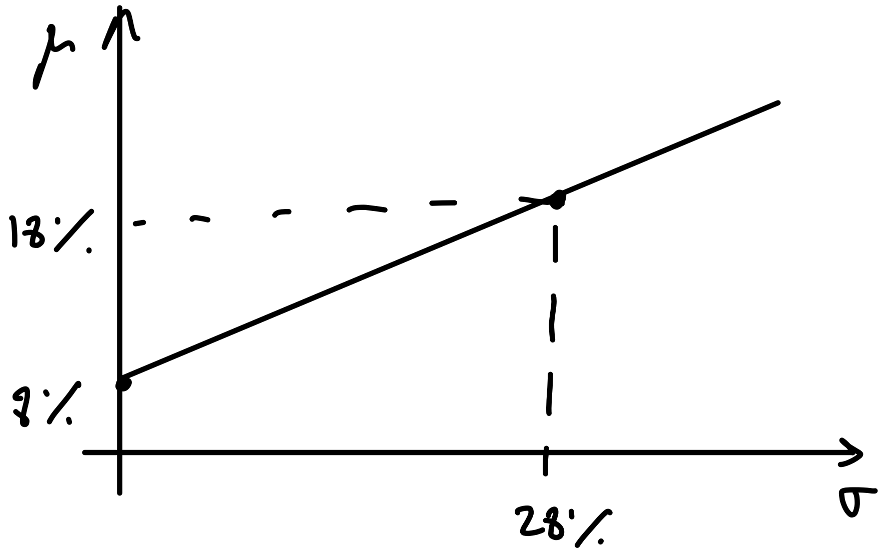
$$W = \frac{0.08 - 0.05}{0.11 - 0.05} = 50\%$$

So 50% in Q and 50% risk-free.

b. $\sigma_P = 0.5 \times 0.15 = 7.5\%$

c. The first client is more risk averse since she prefers a lower standard deviation.

(5)



a. $w^* = \dfrac{0.18 - 0.08}{3.5 \times 0.28^2} = 36.44\%$

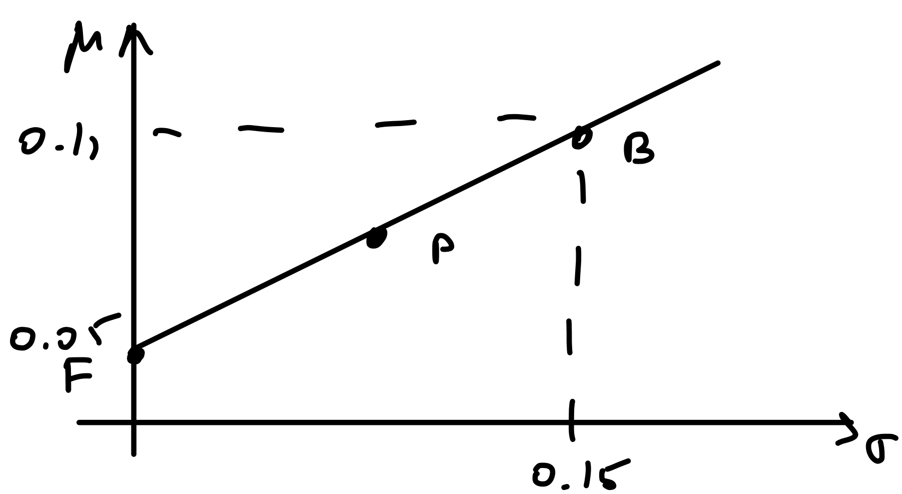
So 36.44% goes into the risky fund.

b. $\mu = (1 - 0.3644) \times 0.08 + 0.3644 \times .18$
$= 11.64\%$

c. $\sigma = 0.3644 \times .28 = 10.20\%$

(6) a. Curve 2 is feasible and provides the greatest level of utility.

b.



$$w^* = \frac{0.11 - 0.05}{A \times 0.15^2} = 0.5$$

$$\Rightarrow A = \frac{0.11 - 0.05}{0.5 \times 0.15^2} = 5.33$$

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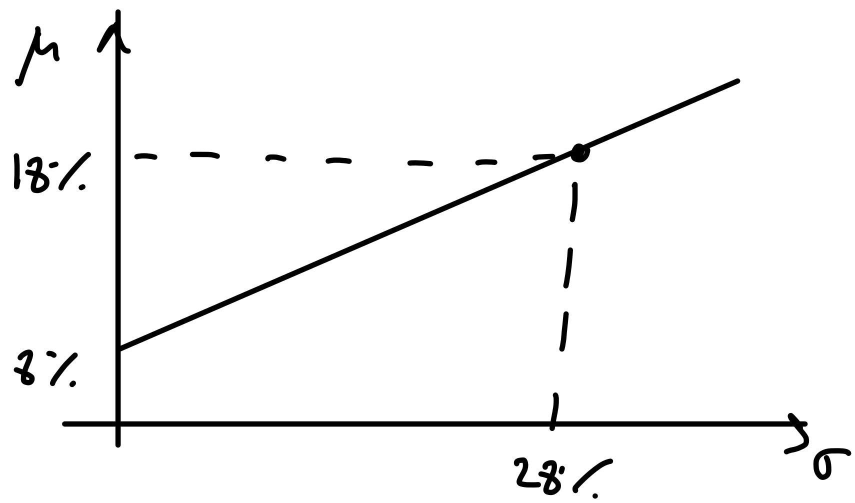


a. $w = 0.7 \qquad 1 - w = 0.3$

$$\mu = 0.3 \times 0.08 + 0.7 \times 0.18 = 15\%$$

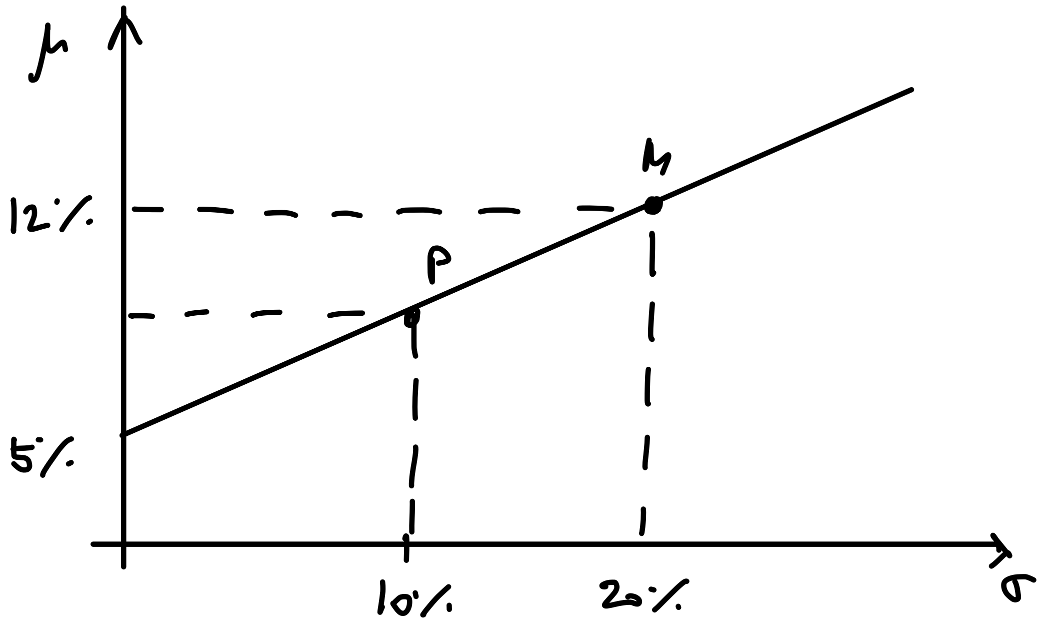
$$\sigma = 0.7 \times 0.28 = 19.6\%$$

b.

Asset	total %
A	17.5%
B	22.4%
C	30.1%
F	30%

c. $SR = \frac{18 - 8}{28} = 0.357.$

(8)



Since $\sigma_p = \frac{1}{2}\sigma_M$

$$\Rightarrow \mu_p = 0.05 + \frac{1}{2}(0.12 - 0.05) = 8.5\%$$

(9) $0.3 = w \; 0.4 \Rightarrow w = 0.75$

$$\mu = 0.25 \times 0.12 + 0.75 \times 0.30$$
$$= 25.5\%$$